Adaptive Synchronization-Based Approach for Parameters Identification in Delayed Chaotic Network

Cui Jun-feng (崔俊峰) *, Yao Hong-xing(姚洪兴) and Shao Hai-jian(邵海见)
Nonlinear Scientific Research Center, Jiangsu University, Zhenjiang Jiangsu, 212013, P.R. China
*Corresponding author *E-mail address*: cjfxxm@yahoo.com.cn

Abstract: In this paper, an adaptive procedure to the problem of synchronization and parameters identification for chaotic networks with time-varying delay is introduced by combining the adaptive control and linear feedback. Especially, we consider that the equations $\dot{x}_i(t)$ (for $i=r+1, r+2,\ldots, n$) can be expressed by the former $\dot{x}_i(t)$ (for $i = 1, 2,\ldots, r$), which is not the same as the previous equation. This approach is also able to track the changes in the operating parameters of the chaotic networks rapidly and the speed of synchronization and parameters estimation can be adjusted. In addition, this method is quite robust against the effect of slight noise and the estimated value of a parameter fluctuates around the correct value.

Keywords: Parameters identification; synchronization- based Time-varying delay; chaotic network

1. Introduction

Chaos appears in time evolutions of some kinds of nonlinear equations [1]. Chaotic systems intrinsically defy synchronization due to the evolution of a chaotic system sensitive dependence on its initial condition. However, over the past few years, the surprising phenomenon of synchronization between coupled chaotic systems has generated much interest since the pioneer work of Pecora and Carrol [2,3]. The interest in understanding the synchronization characteristics of chaotic systems stems from its potential applications in a variety of areas, such as in secure communication, chemical and biological systems, information science, optics and so on [1,4–5]. Recently, a wide variety of approaches have been proposed for the synchronization of chaotic systems, which include PC method[2], OGY method [3], scalar driving method, coupling control, manifold-based method [6], fuzzy control, impulsive control method [7], active control, adaptive control [8–13], and time-delay feedback approach [14], etc. However, the aforementioned approaches and many other synchronization methods are valid for chaotic systems only when the systems' para- -meters are known. But in many practical situations, the values of some systems' parameters cannot be exactly known a priori, and the synchronization will be destroyed and broken with the effects of these uncertainties. Therefore, this paper is devoted to the synchronization-based estimation of connection parameters by combining adaptive scheme and dynamical linear feedback control, provided the model is known. Recently, there is increasing interest in the study of dynamical properties of chaotic networks due to their potential applications in different fields such as combinatorial optimization, pattern recognition, signal, and image processing [15–17]. Most of existing works focused on the stability analysis and periodic oscillation of this kind of chaotic networks. In particular, the introduction of delays into chaotic networks may make their dynamical behaviors much more complicated even with strange attractor [18]. Recently, the chaos synchronization phenomenon for chaotic networks has drawn the attention of some researchers [19,20]. In above Refs. the synchronization schemes are proposed based on exactly knowing the concrete values of the connection weight matrices, and there is an assumption that the parameters in all coupled chaotic networks are identical.

Some synchronization-based strategies have been devised to estimate all unknown parameters of the master chaotic system. In Refs. [10, 21–25], some schemes such as auto-synchronization, random optimization, error minimization and geometric control method have been developed to recover unknown parameter values of a given model. By combining adaptive scheme and dynamical linear feedback, an analytical scheme was proposed for estimating all unknown parameters from time series in Ref. [26-27], based on the invariance principle of differential equations. Moreover, Huang has given a more detailed proof and some interesting remarks on the adaptive-feedback control algorithm [28]. Some other authors have also used synchronization to estimate unknown parameters [29-32, 34]. However, the parameters estimation for delayed chaotic systems has not been explicitly considered and studied.

Motivated by the above discussions, in this paper, we consider the problem of parameters identification for delayed chaotic networks. we extend the method proposed in Refs. [29,30] to the chaotic networks with time-varying delay and show that this method is also effective in this case. In this paper, based on the invariance principle of functional differential equations ，a dynamical feedback process is adopted, which is quite different from conventional feedback scheme. The feedback term vanishes once the synchronization and parameters identification have been achieved successfully.

This paper is organized as follows. In Section 2, the chaotic network model, and some necessary definition, hypotheses are given. In Section 3, by combining the dynamical feedback control and adaptive control, the synchronization-based scheme for parameters identification in chaotic networks with time-varying delay is described. Some numerical examples are given to verify the effectiveness of described parameter estimation scheme in Section 4. We conclude the paper in Section 5.

2. Chaotic Networks Model and Preliminaries

In this section, we consider the following chaotic network:

$$\begin{cases} \dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{r} a_{ij} g_j(x_j(t-\tau)) - \gamma \sum_{j=1}^{r} b_{ij} x_j(t) & i = 1,2,\dots r, \\ \dot{x}_i(t) = k_i(\overline{x}_r(t)) + \sum_{j=r+1}^{n} c_{ij} x_j(t) & i = r+1, r+2, \dots n, \end{cases} \tag{1}$$

Or in a compact form

$$\begin{cases} \dot{x}_r(t) = f(x_r(t)) + Ag(x_r(t-\tau)) - \gamma B x_r(t) \\ \dot{x}_{n-r}(t) = K(x_r(t)) + C x_{n-r}(t) \end{cases} \tag{2}$$

where $x_i(t)(i = 1,2,\dots n)$ denotes the state variable of the chaotic system, $\overline{x}_r(t) = (x_1(t), x_2(t), \dots x_r(t))^T$, r is an integer and $1 \le r \le n, \tau(t) > 0.$ Functions $f_i(\cdot)$ and $g_i(\cdot): R \to R$ are continuous, and $f_i(0) = g_i(0) = 0.$ $\gamma > 0$ is a constant, $k_i(\cdot) \in C[R^r, R]$ and $k_i(0) = 0.$ $A = (a_{ij})_{r\times r}, B = (b_{ij})_{r\times r}$ and $C = (c_{ij})_{(n-r)\times(n-r)}$ are real matrixes and c_{ij} is negative constant, which denote the strength of neuron interconnections.

Throughout the paper, we have the following three assumptions:

(S_1) There exist nonnegative L_i and $\tilde{L}_i (i = 1,2,\dots r)$ such that

$$| f_i(x) - f_i(y) | \le L_i |x - y|, \quad |g_i(x) - g_i(y)| \le \tilde{L}_i |x - y|,$$

For $\forall x, y \in R.$ and let $L = \max_{1\le i \le r} L_i, \tilde{L} = \max_{1\le i\le r} \tilde{L}_{i.}$

(S_2) $\tau(t)$ is a differential function with $0 \le \dot{\tau}(t) < 1$. Clearly, this assumption is certainly ensured if the transmission delay $\tau(t)$ is a constant.

(S_3) For $\forall x = (x_1, x_2, \ldots x_r)^T \in R^r, \|x\|$ denotes the norm of x defined by $\|x\| = (x^T x)^{1/2}$. For $\forall Q \in R^{r\times r}, \|Q\|$ indicates the norm of Q induced by $\|Q\| = (\lambda_{\max}(Q^T Q))^{1/2}$.

Lemma 1. [33] For any vector $x, y \in R^n$, and positive definite $Q \in R^{n\times n}$, the following matrix inequality holds:

$$2x^T y \le x^T Q x + y^T Q^{-1} y.$$

3. Description of the Parameters Identification Scheme

In order to observe the synchronization behavior of system (1), we introduce another chaotic network which is the response system of the drive system (1) . The behavior of the response system depends on the behavior of the drive system, but the drive system is not influenced by the response system.

$$\begin{cases} \dot{y}_i(t) = f_i(y_i(t)) + \sum_{j=1}^{r} \hat{a}_{ij} g_j(y_j(t-\tau)) - \gamma \sum_{j=1}^{r} \hat{b}_{ij} y_j(t) + u_i(t) & i = 1, 2, \ldots r, \\ \dot{y}_i(t) = \hat{k}_i(\bar{y}_r(t)) + \sum_{j=r+1}^{n} \hat{c}_{ij} y_j(t) & i = r+1, r+2, \ldots n, \end{cases} \tag{3}$$

or in a compact form

$$\begin{cases} \dot{y}_r(t) = f(y_r(t)) + \hat{A} g(y_r(t-\tau)) - \gamma \hat{B} y_r(t) - u_r(t) \\ \dot{y}_{n-r}(t) = K(y_r(t)) + \hat{C} y_{n-r}(t) \end{cases} \tag{4}$$

where $y_i(t)(i = 1, 2, \ldots n)$ denotes the state variable of the response system, $u_i(t) = \varepsilon_i \cdot e_i(t)$, $\varepsilon_i = -\alpha_i \cdot e_i^2(t)$ indicates the external control input that will be appropriately designed for a control objective, and $\bar{y}_r(t) = (y_1(t), y_2(t), \ldots y_r(t))^T$.

Let $e_i(t) = x_i(t) - y_i(t)$, the error dynamical system of (1) and (3) is

$$\begin{cases} \dot{e}_i(t) = F_i(e_i(t)) + \sum_{j=1}^{r} a_{ij} G_j(e_j(t-\tau)) + \sum_{j=1}^{r} \tilde{a}_{ij} g_j(y_j(t-\tau)) - \gamma \sum_{j=1}^{r} b_{ij} e_j(t) - \gamma \sum_{j=1}^{r} \tilde{b}_{ij} y_j(t) - u_i(t) & i = 1, 2, \ldots r, \\ \dot{e}_i(t) = K_i(\bar{e}_r(t)) + \sum_{j=r+1}^{n} c_{ij} e_j(t) + \sum_{j=r+1}^{n} \tilde{c}_{ij} y_j(t) & i = r+1, r+2, \ldots n, \end{cases} \tag{5}$$

where $F_i(e_i(t)) = f_i(x_i(t)) - f_i(y_i(t)), G_j(e_i(t-\tau)) = g_j(x_j(t-\tau)) - g_j(y_j(t-\tau)),$

$K_i(\bar{e}_r(t)) = k_i(\bar{x}_r(t)) - k_i(\bar{y}_r(t)).$

Model (5) can be rewritten as the following matrix form

$$\begin{cases} \dot{e}_r(t) = F(e_r(t)) + AG(e_r(t-\tau)) + \tilde{A}g(y_r(t-\tau)) - \gamma Be_r(t) - \gamma\tilde{B}y_r(t) - \varepsilon * e_r(t) \\ \dot{e}_{n-r}(t) = K(e_r(t)) + Ce_{n-r}(t) + \tilde{C}y_{n-r}(t) \end{cases} \tag{6}$$

$\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots \varepsilon_r)^T \in R^r$ are the updated feedback gain; and the mark $*$ is defined as $\varepsilon * e_r(t) = (\varepsilon_1 \cdot e_1(t), \varepsilon_2 \cdot e_2(t), \ldots \varepsilon_r \cdot e_r(t))^T$.

where $e_r(t) = (e_1(t), e_2(t), \ldots e_r(t))^T$, $e_{n-r}(t) = (e_{r+1}(t), e_{r+2}(t), \ldots e_n(t))^T$,

$F(e_r(t)) = (F_1(e_1(t)), F_2(e_2(t)), \ldots F_r(e_r(t)))^T$,

$G(e_r(t-\tau)) = (G_1(e_1(t-\tau)), G_2(e_2(t-\tau)), \ldots G_r(e_r(t-\tau)))^T$, $K(e_r(t)) = (K_{r+1}(e_r(t)), K_{r+2}(e_r(t)), \ldots K_n(e_r(t)))^T$.

Theorem 1. *Under the assumptions* (S_1) , (S_2) *and* (S_3) , *the dynamical feedback strength* $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots \varepsilon_r)^T$ *and the estimated parameters* $\tilde{A}, \tilde{B},$ *and* $\tilde{C}$ *are adapted according to the following updated law, respectively,* $\varepsilon_i = -\alpha_i e_i^2(t), \quad i = 1, 2, \ldots, r,$ *and*

$$\begin{cases} \dot{\hat{a}}_{ij} = -\eta_{ij} e_i(t) g_j(y_j(t-\tau)) \quad i, j = 1, 2, \ldots r \\ \dot{\hat{b}}_{ij} = \gamma\delta_{ij} e_i(t) y_i(t) \qquad i, j = 1, 2, \ldots r \\ \dot{\hat{c}}_{ij} = -\gamma_{ij} e_i(t) y_i(t) \quad i, j = r+1, r+2, \ldots n \end{cases} \tag{7}$$

where $\eta_{ij} > 0, \delta_{ij} > 0, \gamma > 0 (i = 1, 2 \ldots r)$ *and* $\gamma_{ij} > 0 (i = r+1, r+2, \ldots n)$ *are arbitrary constants, respectively, then the controlled response chaotic network* (4), *and satisfies the following condition.*

$$\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} \left| c_{ij} - \hat{c}_{ij} \right| = 0 \qquad i,\ j = r+1,\ r+2, \ldots$$

$$\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} \left| a_{ij} - \hat{a}_{ij} \right| = \lim_{t\to\infty} \left| b_{ij} - \hat{b}_{ij} \right| = 0 \quad i,\ j = 1,\ 2, \ldots$$

Proof. Let $\tilde{A} = A - \hat{A}, \tilde{C} = C - \hat{C}$ and $\tilde{B} = B - \hat{B}$ be the estimation errors of the parameters A, C and B, and subtracting Eq. (2.2) from (3.2). We can yield the error dynamical system as follows:

$$\begin{cases} \dot{e}_r(t) = F(e_r(t)) + AG(e_r(t-\tau)) + \tilde{A}g(y_r(t-\tau)) - \gamma Be_r(t) - \gamma\tilde{B}y_r(t) - \varepsilon * e_r(t) \\ \dot{e}_{n-r}(t) = K(e_r(t)) + Ce_{n-r}(t) + \tilde{C}y_{n-r}(t) \end{cases} \tag{8}$$

$\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots \varepsilon_r)^T \in R^r$ are the updated feedback gain; and the mark $*$ is defined as $\varepsilon * e_r(t) = (\varepsilon_1 \cdot e_1(t), \varepsilon_2 \cdot e_2(t), \ldots \varepsilon_r \cdot e_r(t))^T$

We design the following Lyapunov function:

$$V(e(t))=\frac{1}{2}\sum_{i=1}^{r}e_i^T(t)e_i(t)+\frac{1}{2}\sum_{i=1}^{r}\frac{1}{\alpha_i}(\varepsilon_i-l)^2+\frac{1}{2}\sum_{i=1}^{r}(\sum_{j=1}^{r}\frac{1}{\eta_{ij}}\tilde{a}_{ij}^2+\sum_{j=1}^{r}\frac{1}{\delta_{ij}}\tilde{b}_{ij}^2)$$
$$+\frac{1}{2}\sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\frac{1}{\gamma_{ij}}\tilde{c}_{ij}^2+\frac{1}{2(1-\sigma)}\int_{t-\tau}^{t}g^T(e(s))g(e(s))ds \quad (9)$$

where l is a constant to be determined.

Calculating the derivative of (9) along the trajectories of (8) , we have

$$\dot{V}(e(t))=\sum_{i=1}^{r}e_i^T(t)\dot{e}_i(t)-\sum_{i=1}^{r}(\varepsilon_i-l)\dot{\varepsilon}_i+\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{a}_{ij}\dot{\tilde{a}}_{ij}+\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{b}_{ij}\dot{\tilde{b}}_{ij}+\sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\tilde{c}_{ij}\dot{\tilde{c}}_{ij}$$
$$+\frac{1}{2(1-\sigma)}g^T(e(t))g(e(t))-\frac{1-\dot{\tau}}{2(1-\sigma)}g^T(e(t-\tau))g(e(t-\tau))$$

$$=e_r^T(t)[F(e_r(t))+AG(e_r(t-\tau))+\tilde{A}g(y_r(t-\tau))-\gamma Be_r(t)-\gamma\tilde{B}y_r(t)-\varepsilon*e_r(t)]$$
$$+e_{n-r}^T(t)[K(e_r(t))+Ce_{n-r}(t)+\tilde{C}y_{n-r}(t)]+\sum_{i=1}^{r}(\varepsilon_i-l)e_i^2(t)-\sum_{i=1}^{r}\sum_{j=1}^{r}\tilde{a}_{ij}e_i(t)g_j(y_j(t-\tau))$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}\gamma\tilde{b}_{ij}e_i(t)y_j(t)-\sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\tilde{c}_{ij}e_i(t)y_j(t)+\frac{1}{2(1-\sigma)}g^T(e(t))g(e(t))-\frac{1-\dot{\tau}}{2(1-\sigma)}g^T(e(t-\tau))g(e(t-\tau))$$
$$=e_r^T(t)[F(e_r(t))+AG(e_r(t-\tau))-\gamma Be_r(t)]+e_{n-r}^T(t)[K(e_r(t))+Ce_{n-r}(t)]$$
$$+\sum_{i=1}^{r}-le_i^2(t)+\frac{1}{2(1-\sigma)}g^T(e(t))g(e(t))-\frac{1-\dot{\tau}}{2(1-\sigma)}g^T(e(t-\tau))g(e(t-\tau))$$

According to the properties of (S1) and (S2), then we can get:

$$G^T(e_r(t-\tau))G(e_r(t-\tau))=\sum_{i=1}^{r}G_i^2(e_i(t-\tau))\le\sum_{i=1}^{r}\tilde{L}_i^2e_i^2(t-\tau)=\tilde{L}^2e_r^T(t-\tau)e_r(t-\tau)$$

$$\|F(e_r(t))\|^2\le L^2e_r^T(t)e_r(t),\quad g^T(e(t-\tau))g(e(t-\tau))\ge 0,\frac{1-\dot{\tau}}{2(1-\sigma)}\ge\frac{1}{2}$$

$$g^T(e(t))g(e(t))=\sum_{i=1}^{r}g_i^2(e_i(t))\le\sum_{i=1}^{r}\tilde{L}_i^2e_i^2(t)=\tilde{L}^2e_r^T(t)e_r(t)$$

and by lemma 1, we can obtain

$$e_r^T(t)AG(e_r(t-\tau))\le\frac{1}{2}e_r^T(t)AQ_1\tilde{L}e_r(t)+\frac{1}{2}e_r^T(t-\tau)AQ_1^{-1}\tilde{L}e_r(t-\tau)$$
$$e_{n-r}^T(t)K(e_r(t))\le\frac{1}{2}e_{n-r}^T(t)KQ_2e_{n-r}(t)+\frac{1}{2}e_r^T(t)KQ_2^{-1}e_r(t)$$

As a tool of deriving a less conservative stability criterion, we add the following one zero equation to be chosen as:

$$l_1e_{n-r}^T(t)\times[e_r(t-\tau)-e_r(t-\tau)]=0$$

and by lemma 1, we also have $2e_{n-r}^T(t)e_r(t-\tau) \le e_{n-r}^T(t)Q_3 e_{n-r}(t) + e_r^T(t-\tau)Q_3^{-1}e_r(t-\tau)$

Thus, we will get

$$
\begin{aligned}
\dot{V}(t) \le\ & e_r^T(t)[L-\gamma B - l + \frac{1}{2}(\tilde{L}^2(1-\sigma)^{-1} + KQ_2^{-1} + AQ_1\tilde{L})]e_r(t) \\
& + e_{n-r}^T(t)(C + \frac{KQ_2}{2})e_{n-r}(t) + \frac{1}{2}e_r^T(t-\tau)(-\tilde{L}^2 + AQ_1^{-1})e_r(t-\tau) \\
& + l_1 e_{n-r}^T(t)Q_3 e_{n-r}(t) + l_1 e_r^T(t-\tau)Q_3^{-1}e_r(t-\tau) - l_1 e_{n-r}^T(t)e_r(t-\tau)
\end{aligned}
$$

$$\le \eta^T(t)Q\eta(t) < 0$$

where $\eta(t) = (e_r^T(t), e_{n-r}^T(t), e_r^T(t-\tau))^T$,

$$Q = \begin{pmatrix} \Im_1 & 0 & 0 \\ 0 & \Im_2 & -l_1 I_r \\ 0 & 0 & \Im_3 \end{pmatrix} < 0$$

$$\Im_1 = [L - \gamma B - l + \frac{1}{2}(\tilde{L}^2(1-\sigma)^{-1} + KQ_2^{-1} + AQ_1\tilde{L})]I_r$$

$$\Im_2 = (l_1 Q_3 + C + \frac{KQ_2}{2})I_r, \Im_3 = \frac{1}{2}(-\tilde{L}^2 + AQ_1^{-1} + l_1 Q_3^{-1})I_r$$

Q_i, (i=1,2,3) is positive matrix.

The constant l and l_1 can be properly chosen to make $\dot{V}(t) < 0$

Therefore, based on the lyapunov stability theory, the errors vector $e(t) \to 0$, as $t \to 0$. Then the theorem 1 has been proofed.

It is obvious that $M = \{\dot{V}(e(t)) |= 0\} = \{e(t) = 0\}$. Therefore the set $E = \{e(t) = 0, \hat{C} = C,$ $\hat{A} = A, \hat{B} = B, \varepsilon = \varepsilon_0\}$ is the largest invariant set contained in M for system (8). In fact, if one of the following equalities cannot hold: $\hat{C} = C, \hat{A} = A, \hat{B} = B,$ then $e(t) = 0$ can not be a fixed point of Eq. (8) at all, i.e., one cannot conclude that the solution $e(t)$ is equal to 0 for $t > 0$ when the initial values $e(t) = 0$. So according to the invariant principle of functional differential equations , starting with arbitrary initial values of Eq. (8), the trajectory converges asymptotically to the set E , i.e., $e(t) = 0$, $\hat{C} = C, \hat{A} = A$ and $\hat{B} = B,$ as $t \to \infty$.This indicates that the unknown parameters C, A and B can be successfully estimated using updated laws (5) and (6), and synchronization is achieved at the same time. This ends the proof.

In Theorem 1, we can note that the variable feedback strength ε_i is automatically adapted to a suitable strength depending on the initial values for the synchronization of chaotic networks, which is significantly different from the usual linear feedback. Whereas, by using ordinary linear feedback scheme with constant feedback strength, the unknown parameters can also be estimated with certain updated laws. In the following, a corollary will be given to show the fact.

For simplicity, we assume that all notations are the same as those mentioned before.

Corollary 1. *Under the assumptions* $(S_1),(S_2)$ and (S_3), *the fixed feedback strength* $\varepsilon=(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_r)^T$ *is large enough and the estimated parameters* $\hat{C},\hat{A},$ *and* $\hat{B}$ *are adapted according to the updated law* (7), *then the controlled response chaotic network* (3) *is synchronized with the drive network* (1), *and satisfies the following condition:*

$$\lim_{t\to\infty} e_i(t)=\lim_{t\to\infty}\left|a_{ij}-\hat{a}_{ij}\right|=\lim_{t\to\infty}\left|b_{ij}-\hat{b}_{ij}\right|=0 \quad i,\ j=1,\ 2,\ldots,\ r$$

$$\lim_{t\to\infty} e_i(t)=\lim_{t\to\infty}\left|c_{ij}-\hat{c}_{ij}\right|=0 \quad i,\ j=\ r+1,\ r+2,\ldots,\ n.$$

Proof. By constructing another Lyapunov function:

$$V(e(t))=\frac{1}{2}\sum_{i=1}^{r}e_i^T(t)e_i(t)+\frac{1}{2}\sum_{i=1}^{r}\left(\sum_{j=1}^{r}\frac{1}{\eta_{ij}}\tilde{a}_{ij}^2+\sum_{j=1}^{r}\frac{1}{\delta_{ij}}\tilde{b}_{ij}^2\right)+\frac{1}{2}\sum_{i=r+1}^{n}\sum_{j=r+1}^{n}\frac{1}{\gamma_{ij}}\tilde{c}_{ij}^2 + \frac{1}{2(1-\sigma)}\int_{t-\tau}^{t}g^T(e(s))g(e(s))ds \tag{11}$$

This proof is similar to the proof of Theorem 1, we can easily derive the result. Its proof is straightforward and hence omitted.

Remark 1 The scheme described in this paper can be used to identify the parameters of chaotic systems but not stable systems, because for two stable systems with identical equilibrium point, synchronization can be easily obtained even with completely different parameters and structures. However, as stated in Ref. [30] , the analysis of the results should be based on the LaSalle invariant principle, since the Lyapunov direct method can only guarantee the stability in the sense of Lyapunov but cannot guarantee asymptotic stability.

Remark 2 In practice, the linear feedback controller with large enough strength in Corollary 1 is not realizable. Although appropriate feedback strength can be ascertained by certain calculations for concrete dynamical systems, its value is different for nonidentical systems. Whereas, the feedback strength of different systems can be automatically enhanced to the required value for the synchronization of drive and response systems in Theorem 1.

Remark 3 Some sufficiently large adaptive gains $\alpha_i, \eta_{ij}, \delta_{ij} (i, j = 1, 2, \ldots, r)$ and $\gamma_{ij} (i, j = r+1, \ldots, n)$ would lead to fast synchronization and quick parameters identification, while for sufficiently small adaptive gains, the time to achieve synchronization and parameters estimation may be quite long.

4. An example

In this section, we give a numerical example to demonstrate the effectiveness of our results.

Considering the following system:

$$\begin{cases} \dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{2} a_{ij} g_j(x_j(t-\tau)) - \gamma \sum_{j=1}^{2} b_{ij} x_j(t) \quad i = 1, 2, \\ \dot{x}_3(t) = k_3(\bar{x}_2(t)) + Cx_3(t), \end{cases} \tag{12}$$

where $f_i(x_i(t)) = \tanh(x_i(t)), g_i(x_i(t-\tau)) = \tanh(x_i(t-\tau)), i = 1, 2.$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & \frac{1}{20} \\ \frac{1}{20} & -\frac{1}{10} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $C = -1, k_3(\bar{x}_2(t)) = x_1(t), \gamma = 1$ and $\tau = 1.$ The system satisfies assumption S_1 with $L = \tilde{L} = L_i = \tilde{L}_i = 1, i = 1, 2.$ The initial condition $(x_1(t), x_2(t), x_3(t))^T = (0.2, 0.1, 0.3)^T$, for $t \in [-1, 0]$

The response system of network (3.1) is

$$\begin{cases} \dot{y}_i(t) = f_i(y_i(t)) + \sum_{j=1}^{2} \hat{a}_{ij} g_j(y_j(t-\tau)) - \gamma \sum_{j=1}^{2} \hat{b}_{ij} y_j(t) + u_i(t) \quad i = 1, 2, \\ \dot{y}_3(t) = \hat{k}_i(\bar{y}_2(t)) + Cy_3(t). \end{cases} \tag{13}$$

where $f_i(x_i(t)) = \tanh(x_i(t)), g_i(x_i(t-\tau)) = \tanh(x_i(t-\tau)), i = 1, 2.$

$\hat{A} = \begin{pmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & \frac{3}{20} \\ \frac{3}{20} & -\frac{3}{10} \end{pmatrix}, \hat{B} = \begin{pmatrix} \hat{b}_{11} & \hat{b}_{12} \\ \hat{b}_{21} & \hat{b}_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $C = -1, k_3(\bar{x}_2(t)) = x_1(t), \gamma = 1$ and $\tau = 1.$ The system satisfies assumption S_1 with $L = \tilde{L} = L_i = \tilde{L}_i = 1, i = 1, 2.$ The initial condition $(y_1(t), y_2(t), y_3(t))^T = (0.4, 0.2, 0.6)^T$, for $t \in [-1, 0]$;

$\varepsilon_i(0) = \hat{a}_{ii}(0) = \hat{b}_{ii}(0) = 0, i = 1, 2,$ and parameters update gain $\eta_{ij} = 10, .\ \delta_{ij} = 10, (i, j = 1, 2),\ \gamma_{33} = 10$. Let $e_i(t) = x_i(t) - y_i(t),$ the error dynamical system of (12) and (13) is

$$\begin{cases} \dot{e}_i(t) = F_i(e_i(t)) + \sum_{j=1}^{2} a_{ij} G_j(e_j(t-\tau)) + \sum_{j=1}^{2} \tilde{a}_{ij} g_j(y_j(t-\tau)) - \gamma \sum_{j=1}^{2} b_{ij} e_j(t) - \gamma \sum_{j=1}^{2} \tilde{b}_{ij} y_j(t) - u_i(t) \quad i=1,2, \\ \dot{e}_3(t) = K_3(\bar{e}_2(t)) + Ce_3(t) + \tilde{C} y_3(t) \end{cases} \tag{14}$$

where $F_i(e_i(t)) = f_i(x_i(t)) - f_i(y_i(t)), G_j(e_i(t-\tau)) = g_j(x_j(t-\tau)) - g_j(y_j(t-\tau)),\ i=1,2,$

$K_3(\bar{e}_r(t)) = k_3(\bar{x}_2(t)) - k_3(\bar{y}_2(t)),$ and $\bar{e}_2(t) = (e_1(t), e_2(t))^T$.

5. Conclusion

In summary, we have shown that a combination of synchronization based on dynamical feedback with an adaptive evolution for parameters unknown to the responser, enables the estimation of the unknown parameters for uncertain delayed chaotic network. In comparison with previous methods, time-delay is taken into account in this simple, analytical and systematic synchronization-based parameters identification scheme. In addition, it is quite robust against the effect of noise, and it is also able to rapidly track changes in the operating parameters of the experimental chaotic network. We also believe that this approach can be easily generalized to the case of other continuous and discrete time-delayed chaotic networks, and also the case of other chaotic dynamical systems with delay.

Acknowledgment

Project supported by the National Natural Science Foundations of China (Grant Nos 70871056), the Society Science Foundation from Ministry of Education of China (Grant 08JA790057) and the Advanced Talents' Foundation and Student's Foundation of Jiangsu University (Nos 07JDG054 and 07A075).

References

[1]Chen G R and Dong X 1998 *from Chaos to Order: Methodologies, Perspectives, and Applications* (Singapore: World Science Publishing Company)
[2]Pecora L M and Carroll T L 1990 *Phys. Rev. Lett*. **64** 821
[3]Ott E, Grebogi C and Yorke J A 1990 *Phys. Rev. Lett*. **64** 1196
[4]Ojalvo J G.and Roy R 2001 *Phys. Rev. Lett.* **22** 5204
[5]Zhao H Q and Zhang J S 2008 *Chinese Physics B* **8** 3996
[6]Yu X H, Chen G.R, Xia Y, Song Y X and Cao Z W 2001 *Circuits Syst*. I **8** 930
[7]P Li, Cao J D, Wang Z D 2007 *Physica A* **373** 261
[8]Xiong W J, Xie W and Cao J D 2006 *Physica A* **370** 832
[9]Cai L L and Nong L 2008 *Chinese Physics B* **12** 7500
[10]Huang D B 2005 *Phys. Rev. E* 71
[11]Li X C, Xiao Y Z and Xu W 2008 *Chinese Physics B* **8** 4721
[12]Huang D B 2004 *Phys. Rev. Lett.* 93
[13]Lei Y M and X W 2008 *Chinese Physics B* **6** 3342
[14]Cao J D, Li H X and Ho D W C 2005 *Chaos Solitons Fractals* **23** 1285
[15]Xu S Y, Lam J, Ho D W C and Zou Y 2005 *IEEE Trans.Circuits Syst.* **6** 349
[16] Chen B C, Tian Q and Zhang Y 2007 *Chinese Physics B* **1** 56
[17]Liao X X, Wang J, and Zeng Z G 2005 *IEEE Trans. Circuits Syst*.**7** 403
[18]Lu H T 2002 *Phys. Lett.* **298** 109
[19]Chen G R, Zhou J and Liu Z R 2004 *Chaos* **7** 2229
[20]Cheng C J, Liao T L and Hwang C C 2005 *Chaos Solitons Fractals* **24** 197
[21]Cao J D and Lu J Q 2006 *Chaos* 16
[22]Yu D C and Wu A 2005 *Phys. Rev. Lett.* 94
[23] Hai P, Wang X D and Yang Y X 2007 *Chinese Physics B* **1** 51
[24]Freitas U S, Macau E E N and Grebogi C 2005 *Phys. Rev. E* 71
[25] Wang X Y and Wu X J 2006 *Chinese Physics B* **2** 605
[26]Konnur R 2003 *Phys. Rev. E* 67
[27]Huang D B 2004 *Phys. Rev. E* 69
[28]Huang D B 2006 *Phys. Rev. E* 73

[29]Li S, Li R H and Xu W 2006 *Chinese Physics* **2** 598

[30]Lu J A, Tao C H, Lu J H and Liu M 2002 *Phys. Lett.* **5** 632

[31]Lu J Q and Cao J D 2007 *Phys. A* **382** 672

[32]Xiong W J, Xie W and Cao J D 2006 *Phys. A* **370** 832

[33] Lu J and Cao J 2007 *Physica A* **382** 672

[34] Cai G L and Shao H J 2009 *Chinese Physics B* **19** 060507